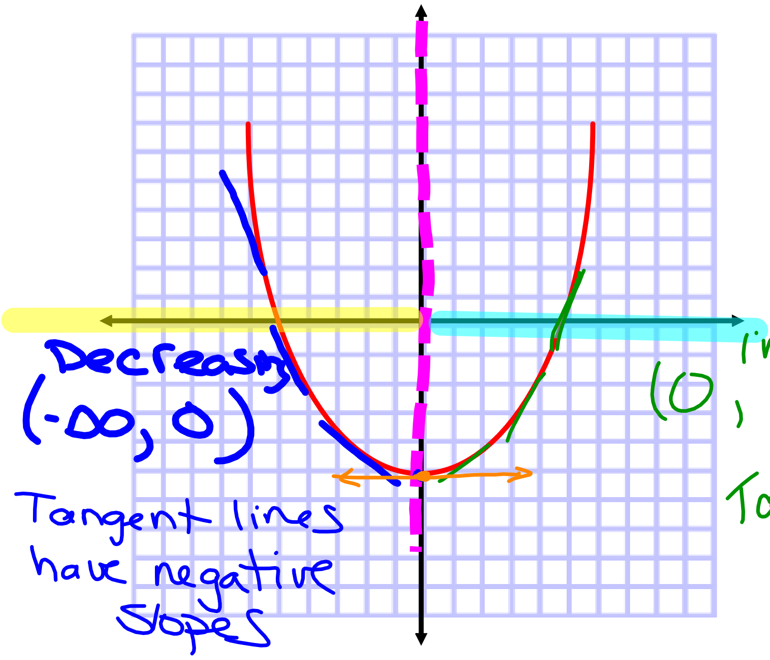
Increasing and Decreasing Functions

and the First Derivative Test

Increasing and Decreasing



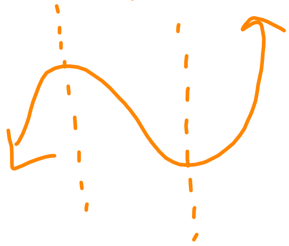
Decreasing
$(-\infty, 0)$

Tangent lines have negative slopes

increasing
$(0, \infty)$

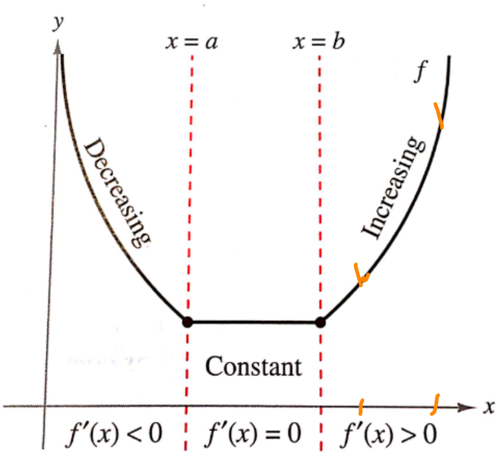
Tangent lines are positive

$(-\infty, 5) \cup (15, \infty)$



Derivatives can be used to classify relative extrema.

Let f be a function that is continuous on the closed interval [a,b] and differentiable on the open interval (a,b).



1. If f'(x) >0 for all x in (a,b), then f is increasing on [a,b]

2. If f'(x)<0 for all x in (a,b), then f is decreasing on [a,b]

3. If f'(x)=0 for all x in (a,b), then f is constant on [a,b]

Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing.

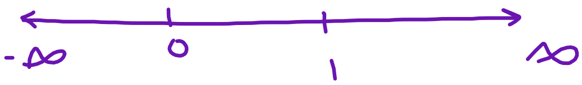
$f'(x) = 3x^2 - 3x$

$3x^2 - 3x = 0$

$3x(x-1) = 0$

$3x = 0 \quad x - 1 = 0$

$x = 0, 1$



① f '(x)

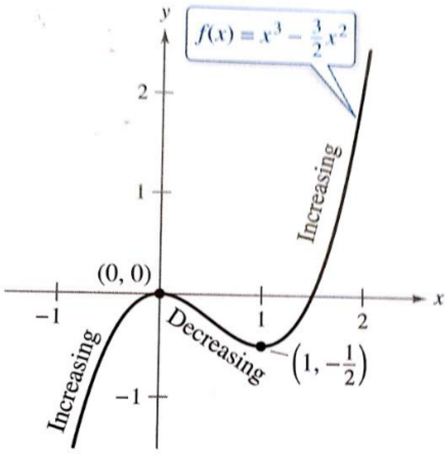
② Critical Numbers

③ Use the CN to set up the intervals

Interval	$(-\infty, 0)$ $-\infty < x < 0$	$(0, 1)$ $0 < x < 1$	$(1, \infty)$ $1 < x < \infty$
Test Value choose # inside the interval	-1	$\frac{2}{3}$	6
Sign of f'(x)	$3x^2 - 3x$ $+$ $3(-1)^2 - 3(-1)$	$-$	$+$
Conclusion	incr	Dec	Incr

$\therefore$ Incr : $(-\infty, 0) \cup (1, \infty)$

Decr : $(0, 1)$



$f(x) = x^3$

$f'(x) = 3x^2$

$3x^2 = 0$

$x = 0$

$(-\infty, 0)$ $(0, \infty)$

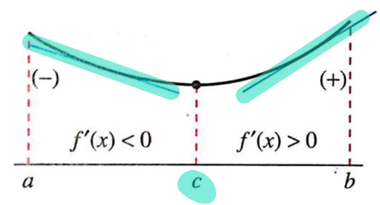
-1 2

$+$ $+$

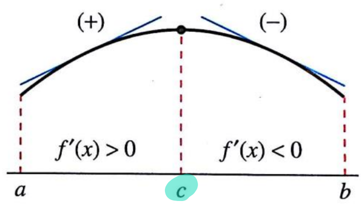
THEOREM 3.6 The First Derivative Test

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

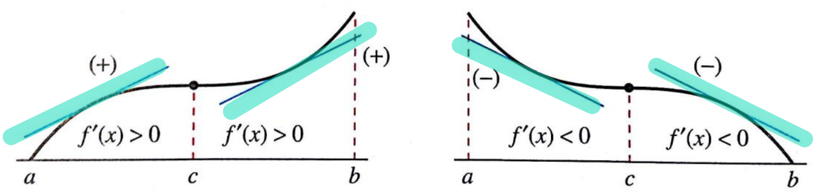
1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.



Relative minimum



Relative maximum



Neither relative minimum nor relative maximum

Find the relative extrema of $f(x)=\frac{1}{2}x-\sin x$

$f'(x)=\frac{1}{2}-\cos x$

$\frac{1}{2}-\cos x=0$

$-\cos x=-\frac{1}{2}$

$\cos x=\frac{1}{2}$

$x=\frac{\pi}{3},\ \frac{5\pi}{3}$

$(0, \pi/3)$	$(\pi/3, 5\pi/3)$	$(5\pi/3, 2\pi)$	
$\pi/6$	π	$11\pi/6$	
$-$	$+$	$-$	
Decr	Incr	Decr	

min $\left(\frac{\pi}{3}, \frac{\pi-3\sqrt{3}}{6}\right)$ max $\left(\frac{5\pi}{3}, \frac{5\pi+3\sqrt{3}}{6}\right)$

Find the relative extrema of $f(x)=\left(x^2-4\right)^{\frac{2}{3}}$

$f'(x) = \frac{2}{3}(x^2-4)^{-\frac{1}{3}}(2x)$

$f'(x) = \frac{4x}{3(x^2-4)^{1/3}}$

CN

$\frac{4x}{3(x^2-4)^{1/3}} = 0$

$x=0$

undefined

$3(x^2-4)^{1/3} = 0$

$x = \pm 2$

$(-\infty,-2)$	$(-2,0)$	$(0,2)$	$(2,\infty)$	intervals
-3	-1	1	3	Test value
$-$	$+$	$-$	$+$	$f'(x)$
Decr	Incr	Decr	Incr	conclusion

min $(-2,0)$, $(2,0)$

Max $(0, \sqrt[3]{16})$